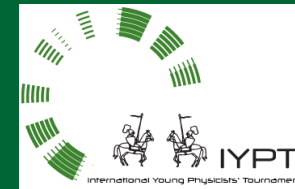




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## 11. Balls on an Elastic Band

Connect two metal balls with an elastic band, then twist the elastic band and put the balls on a table.

The balls will begin to spin in one direction, then in the other. Explain this phenomenon and investigate how the behaviour of such a "pendulum" depends on the relevant parameters.

## 11. Guličky na elastickom páse

Spojte dve kovové guličky elastickým pásom, potom pás zatočte a položte guličky na stôl. Budú sa točiť najskôr do jednej strany a potom do druhej. Vysvetlite tento jav a preskúmajte, ako správanie takéhoto „kyvadla“ závisí od relevantných parametrov.



# 11. Balls on an Elastic Band

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**two metal balls**

**elastic band**

**twist the elastic band**

**balls on a table**

**balls will spin in one direction  
then in the other**





# 11. Balls on an Elastic Band

Connect two metal balls with an elastic band, then twist the elastic band and put the balls on a table.

The balls will begin to spin in one direction, then in the other. Explain this phenomenon and investigate how the behaviour of such a "pendulum" depends on the relevant parameters.

**Explain phenomenon**

**Investigate**

how the **behaviour** of a "pendulum"

**depends**

on the **relevant parameters**



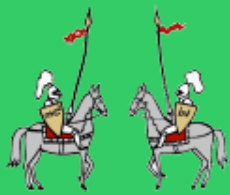


# 11. Balls on an Elastic Band

## How does it work?

- the twisted and shortened elastic band spins
- the balls are spinning
- the band causes circular motion
- the centrifugal force acts on the balls
- the unwinded bend is elongated
- the band exerts a force until it spins
- due to the inertia of the balls, they wind the band in opposite direction
- twisting takes energy away from the balls until they stop
- the twisted band spins ball again...





# 11. Balls on an Elastic Band

## Basic physics

### circular motion

$$F = ma_t$$

$$\omega = \frac{d\varphi}{dt} \quad \varepsilon = \frac{d\omega}{dt} \quad a_t = r\varepsilon$$

$\omega$  – angular velocity,

$\varepsilon$  – angular acceleration

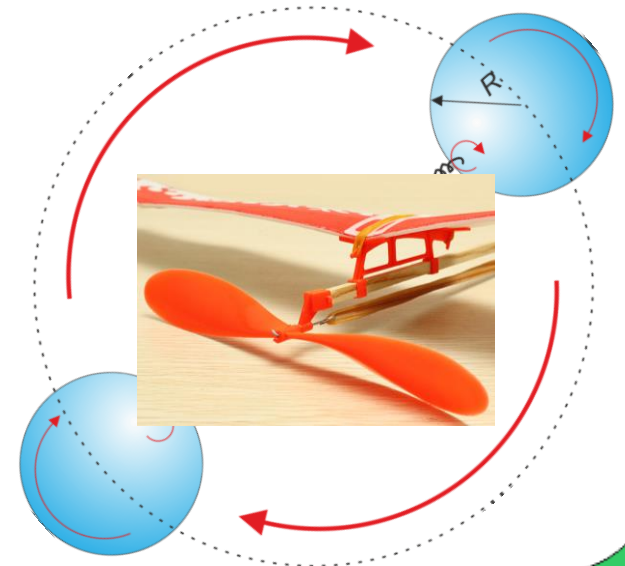
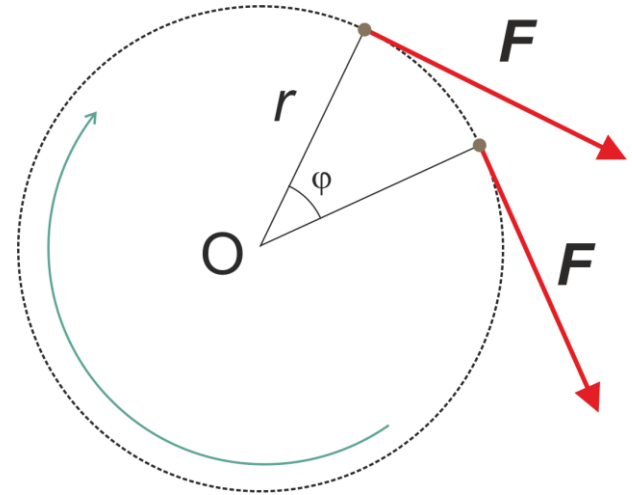
$$Fr = mr^2\varepsilon$$

$$I = mr^2 \quad M = Fr$$

$I$  – moment of inertia,  $M$  – torque (moment of force)

$$M = I\varepsilon$$

- **the twisted belt acts with a torque**  
(similar to the rubber drive of the aircraft model)





# 11. Balls on an Elastic Band

## Basic physics

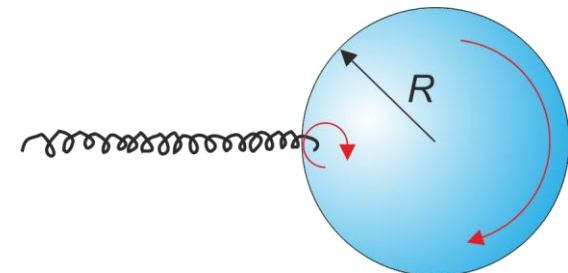
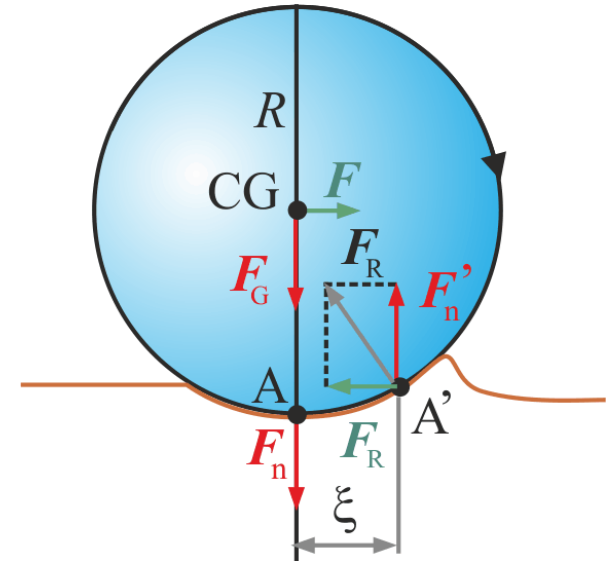
### Rolling ball

- balls are at the ends of the bend
- action - reaction forces from band
- rolling friction

$$F_R = \frac{\xi F_n}{R}$$

- the ball rolls on the table, without slipping
- static friction prevents slipping

$$F_S = \mu F_n$$





# 11. Balls on an Elastic Band

## Basic physics

moment of inertia

$$I_{CG} = \frac{2}{5}mR^2$$

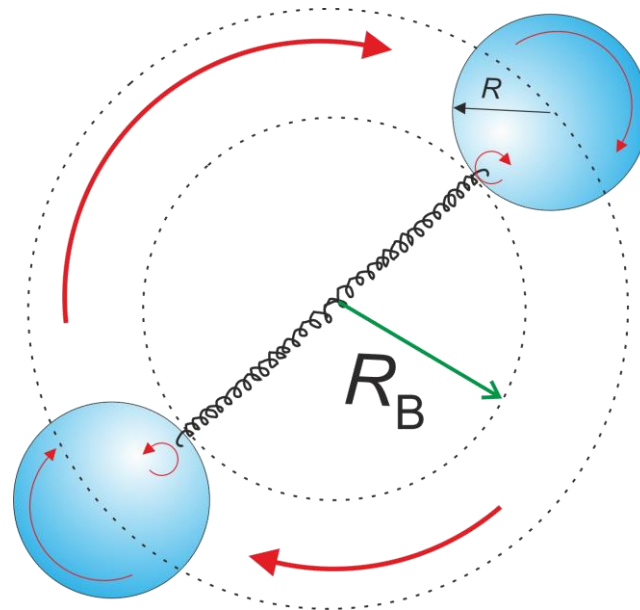
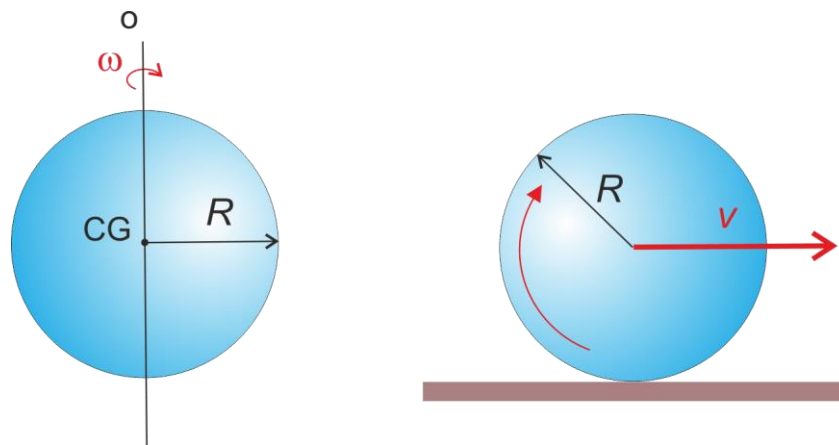
kinetic energy of ball

$$E_K = \frac{1}{2}mv_0^2 + \frac{1}{2}I\omega^2$$

centripetal force, circular motion

$$v = \omega(R_B + R) \quad F_C = m \frac{v^2}{(R_B + R)}$$

$$F_C = m\omega^2(R_B + R)$$





# 11. Balls on an Elastic Band

## Basic physics

- rotation of two balls around its center of gravity (flywheel)

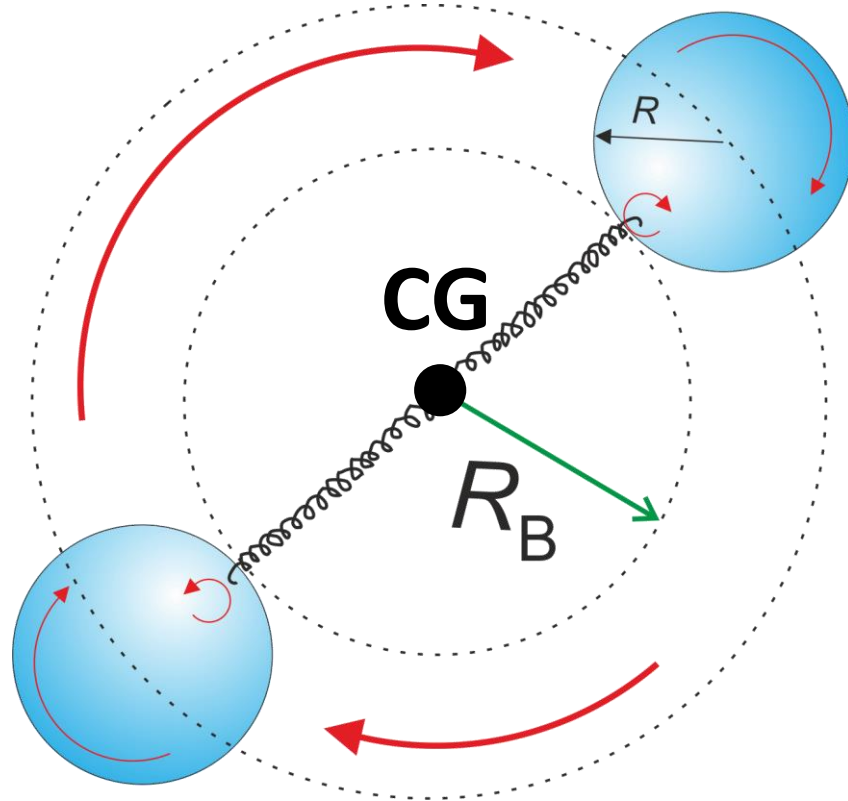
$$E_K = \frac{1}{2} 2m(R_B + R)^2 \omega^2$$

- own balls rotation

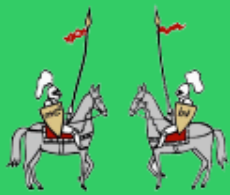
$$E_K = \frac{12}{25} 2mR^2 \omega^2$$

- pendulum

$$F = -ky$$







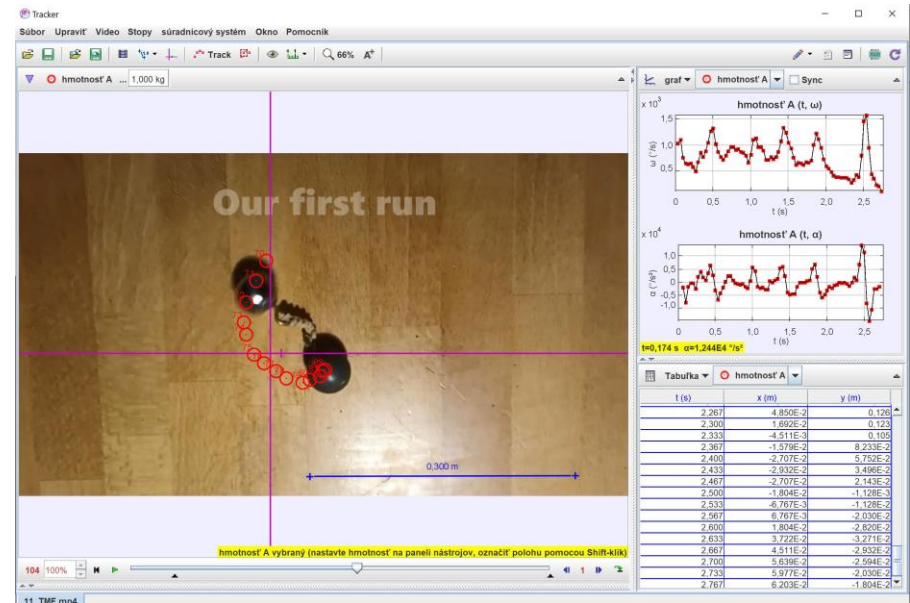
# 11. Balls on an Elastic Band

## Behaviour of pendulum

- time dependence of angular velocity (acceleration)
- changing of band length
- period and frequency of oscillation
- energy and dumping of oscillation
- precession
- ...



<https://physlets.org/tracker/>





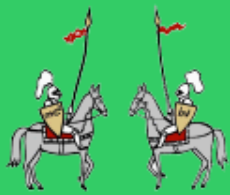
# 11. Balls on an Elastic Band

## „Relevant“ parameters

consider

- different balls (mass, diameter, moment of inertia – hollow/full)
- elastic band (elasticity, length, number of turns)
- surface (coefficient of friction, (not) perfectly horizontal)
- ...





# 11. Balls on an Elastic Band

## Possible approaches to the task

1. How much energy we put into band?
2. How does the band release energy? (IYPT 2014, 14. Rubber motor)
3. Observation and description of different ball oscillations.
4. Explanation of the phenomenon and theoretical model of oscillation.
5. Investigation of the influence of parameters on the pendulum behavior.
6. Comparison of model and experimental results.



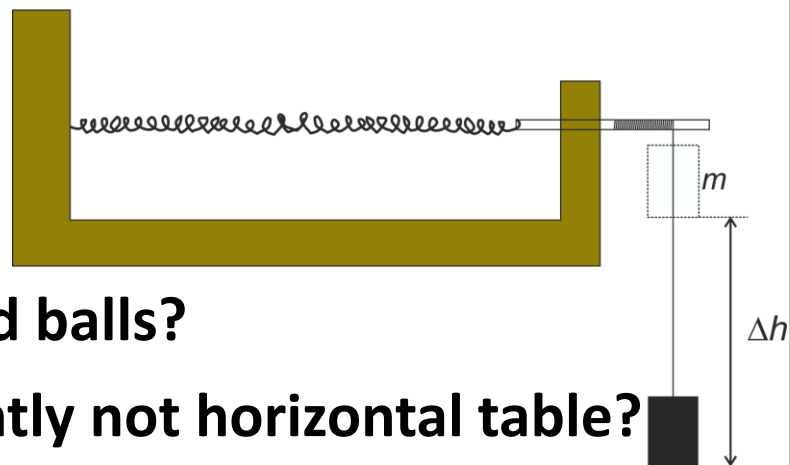
# 11. Balls on an Elastic Band

## Ideas for experimentation

How much energy we put into band?

How pendulum works with different sized balls?

How does a pendulum behave on a gently not horizontal table?





# 11. Balls on an Elastic Band

## Set up





# 11. Balls on an Elastic Band

**Enjoy the fun with the task No 11**

**Balls on an Elastic Band**